Problem 1: Fixed Income

1) Compute the annual total cash flow payments for each bond.

Bond	Time to maturity	Coupon	Price	Note
Bullet	1 year	2,00%	101,0	
Serial	2 years	4,00%	104,0	
Annuity	3 years	4,00%	104,5	36,035% annual payments

Face value 100

1Y Bullet 2,00%	1
Interest	2,00
Principal reduction	100,00
Total payment	102,00

In general interest payment = (Face value - cumulative principal reduction) * Coupon rate

2Y Serial 4,00%	1	2
Interest	4,00	2,00
Principal reduction	50,00	50,00
Total payment	54,00	52,00

Principal reduction = 100/2

3Y Annuity 4,00%	1	2	3
Interest	4,00	2,72	1,39
Principal reduction	32,03	33,32	34,65
Total payment	36,03	36,03	36,03

Annual payments = 36.035% * 100

2) Calculate the corresponding 1, 2 and 3 year zero-coupon bond discount rates and zero-coupon bond yields. Calculate the 1 year zero-coupon forward rates.

$$\boldsymbol{\pi} = \mathbf{Cd}$$
$$\Updownarrow$$
$$\mathbf{d} = (\mathbf{C})^{-1}\boldsymbol{\pi}$$

$$y_t = \left(\frac{1}{d_t}\right)^{\frac{1}{t}} - 1 \qquad f_t = \left(\frac{d_t}{d_{t+1}}\right) - 1$$

π	C		
101	102,00	0,00	0,00
104	54,00	52,00	0,00
104,5	36,03	36,03	36,03

t	d	y	f
1	0,990	1,0%	1,9%
2	0,972	1,4%	3,6%
3	0,938	2,2%	

3) How is Macaulay duration defined and what are the two interpretations of the duration measure?

Macaulay duration is based on the assumption of a flat term structure and is only concerned with parallel shifts in the term structure.
It is defined as

(A)
$$D = \frac{1}{PV(C,y)} \sum_{t=1}^{T} t \frac{c_t}{(1+y)^t}$$

or

(B)
$$D = \sum_{t=1}^{T} t w_t, \text{ where } w_t = \frac{c_t}{(1+y)^t} / \frac{1}{PV(C,y)}$$

Interpretation:
1: Duration (D) is a bond's interest rate risk. A 1% interest rate increase/decrease causes an approximately D% price decrease/increase.
2: Duration can also be interpreted as the length of time that a bond can ensure an average annual return equal to the bond's rate r.

4) Calculate the Macaulay duration and the modified duration on the 3 year annuity bond.

Yield to maturity (and term structure) has been assumed to be 2,50%

Macaulay duration:

		w	t
3Y ann PV =	102,92		
PV(c1)	35,16	34%	1
PV(c2)	34,30	33%	2
PV(c3)	33,46	33%	3

Macaulay duration is $\sum t \times w(t)$ = 1,984
Modified duration = Macaulay duration / (1 + y) = 1,935

5) Assume a 125 basis point parallel downward shift in the yield curve. What is the approximate %-change in the price of the 3 year annuity bond as estimated by the modified duration measure?

$$\frac{\Delta PV(c,y)}{PV(c,y)} = -D_{MOD} \cdot \Delta y$$ Therefore

%-change = -1,935 × -1,250 = 2,419% from a 125 bp downward shift in interest rate.

6) Calculate the exact %-change in the annuity's price. What causes our estimation error from using modified duration in question 5?

PV at 2.50% interest rate =	102,92
PV at 1.25% (2.50% - 1.25%) int rate =	105,46
%-change = PV(1.25%)/PV(2.50%)-1=	2,469%

The duration estimate of the annuity's price change only captures first order effects. Therefore the annuity's convexity (2nd order price effect) or higher order effects are missing out from the simple modified duration estimate of the price change from interest rate movements.

Problem 2: Mean Variance and CAPM

1) What is the expected return, variance and standard deviation on the two asset classes?

(NOTE! Do not correct for degrees of freedom. For instance find σ2 rather than s2)

	Returns		R-E(R)		Squared errors	
Year	**Europe**	**EM**	**Europe**	**EM**	**Europe**	**EM**
2004	12,2%	16,1%	7,54%	2,66%	0,006	0,001
2005	24,9%	35,3%	20,24%	21,86%	0,041	0,048
2006	19,1%	28,5%	14,44%	15,06%	0,021	0,023
2007	6,0%	33,2%	1,34%	19,76%	0,000	0,039
2008	-38,9%	-45,9%	-43,56%	-59,34%	0,190	0,352
			Variance (Sum Sq Err/5)		0,0515	0,0925

Take the probability weighted average of returns and sum of squared errors to find expected return and variance. Standard deviation is the square root of variance
In the case of historical observations prob = 1/N = 0.20
E(R) = ∑prob × R
Var(R) = ∑prob × (R-E(R))

	E(R)	Var(R)	Std dev
Europe	4,7%	0,051	22,7%
EM	13,4%	0,092	30,4%

2) Calculate the covariance and correlation between the two asset class returns and list the covariance matrix.

$$\sigma_{12} = E\left[(\tilde{r}_1 - \bar{r}_1)(\tilde{r}_2 - \bar{r}_2)\right] = Cov(\tilde{r}_1, \tilde{r}_2)$$

$$\rho_{12} = \frac{Cov(\tilde{r}_1, \tilde{r}_2)}{\sigma_1 \sigma_2}$$

	Returns		R-E(R)		Squared errors		Product of errors
Year	**Europe**	**EM**	**Europe**	**EM**	**Europe**	**EM**	
2004	12,2%	16,1%	7,5%	2,7%	0,6%	0,1%	0,002
2005	24,9%	35,3%	20,2%	21,9%	4,1%	4,8%	0,044
2006	19,1%	28,5%	14,4%	15,1%	2,1%	2,3%	0,022
2007	6,0%	33,2%	1,3%	19,8%	0,0%	3,9%	0,003
2008	-38,9%	-45,9%	-43,6%	-59,3%	19,0%	35,2%	0,258
				Covariance (Sum product of err/5)			0,065826

Covariance(Europe, EM)	0,065826
Correlation(Europe, EM)	0,9540169

Covariance matrix:

	Europe	EM
Europe	0,051	0,066
EM	0,066	0,092

3) Assume that you create a portfolio that consists of 20% European Equities and 80% Emerging Market Equities. What is the expected return, variance and standard deviation of the portfolio?

$$E(\tilde{R}_p) = E(x_1\tilde{r}_1 + x_2\tilde{r}_2 + \cdots + x_N\tilde{r}_N)$$
$$= x_1E(\tilde{r}_1) + x_2E(\tilde{r}_2) + \cdots + x_NE(\tilde{r}_N)$$
$$Var(\tilde{R}_p) = Var(x_1\tilde{r}_1 + x_2\tilde{r}_3 + \cdots) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

x(Europe) =	20%	
x(EM)=	80%	
E(Europe) =	4,7%	
E(EM) =	13,4%	
Cov(EM,Europe) =		0,066
E(R) =	11,7%	
Var(R) =	0,0823	
Std dev =	28,7%	

4) What is required of the correlation between any two assets in a long only portfolio in order to have even the slightest systematic risk diversification benefits? What is required of the correlation in order to eliminate all systematic risk and what return should be expected of a zero risk portfolio assuming no arbitrage?

We are in a market with no shorting allowed.
For any correlation between any two assets of less than 1 (perfect correlation) we may obtain diversification benefits on systematic risk. In a long only porfolio (i.e. no negative asset weights) we may only remove all systematic risk if the the correlation between two assets is = -1 (perfect negative correlation).
In this case a zero systematic risk portfolio has an expected return = risk free rate

5) What is the beta of Emerging Market Equities? What kind of risk-sensitivity does beta measure? What is the interpretation of the riskiness of Emerging Market Equities based on the found beta?

$$\beta_i \equiv \frac{Cov(\tilde{r}_i, \tilde{R}_T)}{Var(\tilde{R}_T)}$$

Risk free rate =	2%
Covariance(EM,Market) =	0,08
E(Market return) =	6,1%
Std dev(Market) =	25%
Var(Market) = Std dev(Market)^2 =	0,0625

Beta(EM) = 1,28

Beta measures an asset's systematic risk sensitivity. In the CAPM model it measures the asset's systematic risk relative to the market portfolio.

EM's beta indicates that EM is a relatively risky stock compared to the market risk.

6) What is the expected return on Emerging Market Equities according to the CAPM model?

Expected return(EM) = risk free + beta(EM) × (E(Market return) - risk free rate) = 7,4%

7) Say the CAPM model's prediction about the expected return on Emerging Market Equities is accurate. Do the analysts then over- or undervalue Emerging Market Equities? Why?

According to the analysts' predictions the expected return on EM is ((29 + 0.2)/27) -1 = 8,1%

Since CAPM predicts a return of 7,4%, the analysts are overvaluing the index relative to what CAPM predicts.

Problem 3: Options

1) Estimate the zero-cost forward price of the forward (hint: use continuous compounding)

$F_0 = S_0 e^{r_f T} = 82,58$

2) Estimate the price of the European call option.

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = 0,36$$

$d_2 = d_1 - \sigma\sqrt{T} = 0,22$
$N(d_1) = 0,641$
$N(d_2) = 0,587$
$c_0 = S_0 N(d_1) - K e^{-r_f T} N(d_2) = 2,84$

3) Estimate the price of a comparable European put option.

Use put-call parity in continous time:
$p_0 = c_0 - S_0 + K e^{-r_f T} = 5,09$

4) Comment briefly on how the price of the European call would be related to the price of the European put if their strike price was similar to the zero-cost forward price in question 1.

Based on the put-call parity they should be identical.

5) Comment on the possibility of arbitrage if another European call option with identical expiration is trading with another implied volatility.

Arbitrage is possible as at least one of the options can be synthetically created by a tracking portfolio with a different price.

6) Estimate the price of the European call option when Candyman & Co. pays a dividend

Strip the share price of dividends.
$S_0^* = S_0 - Div \times e^{-r_f T} = 67,68$

$$d_1^* = \frac{\ln\left(\frac{S_0^*}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = -0,55$$

$d_2^* = d_1^* - \sigma\sqrt{T} = -0,69$
$N(d_1^*) = 0,291$
$N(d_2^*) = 0,244$

$c_0^* = S_0^* N(d_1^*) - K e^{-r_f T} N(d_2^*) = 0,32$

The price of the call is lower than in question 1 as expected.

Problem 4: Essay Questions

1) Define the concept of real options and explain how they add value to an investment

A real option is a strategic option to adjust the operations of a firm given the state of the economy.

A real option can be one of the following:

- Shrink, abandon, or temporary shutdown a project a project
  - Contract production if it generates losses
- Wait and invest later instead of investing now
  - Invest when the NPV is the highest
- Vary the mix of the firm’s output or production methods
  - Adjust production based on the cheapest input and the output which provides the highest profit
- Expand or make follow-up investments
  - Creates value by expanding production when prices and/or demand is high or initiating follow-up projects only when they are profitable

2) Explain whether there are tax advantages to leasing

Assume a tax exempt and tax-paying investor exists:

The ownership of an asset has the associated costs for a tax exempt institution:

- Costs of owning = debt repayment + interest

In a competitive market the income of leasing out an asset must equal the cost of owning the asset (the owner is the taxpaying investor)

- Lease payment (1 – T_C) = debt repayment + interest (1 – T_C) – (depreciation deduction)T_C
- Lease payment = (debt repayment – (depreciation deduction)T_C)/(1 – T_C) + interest

If the depreciation deduction exceeds the debt repayment it will be cheaper for tax exempt investors to lease than buy the asset

(Over the lifetime of the asset the total debt repayment will equal the total depreciation deduction. Hence, leasing is advantageous if depreciation exceeds the debt repayment in the early years of the lease)

3) Discuss the differences between direct and indirect bankruptcy costs.

Direct: Costs associated with the occurrence of bankruptcy

- Time spend dealing with creditors – opportunity costs
- Fees to lawyers, investment bankers and others who are involved in the process of gathering and distributing the proceeds

Indirect: Costs associated with a firm making suboptimal operating decisions due to the risk of bankruptcy – it does not maximize the enterprise value (EV) / value of the firm

- Debt-equity holder conflict
    - Debt overhang
    - Asset substitution
    - Shortsighted investment
    - Reluctance to liquidate